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On asymmetries in optics: fundamental considerations and some current applications

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The photon, as the basic constituent of light, is presented as example of a fundamental brick of the physical World which breaks a symmetry. Its properties are illustrated in a heuristic way, to render the discussion accessible to non-specialists, together with the consequences of combining more photons. At the macroscopic scale, when large number of photons are involved – such as in the light coming from the sun – the symmetry is restored through the simultaneous participation of large ensembles of photons. However, we show that macroscopic devices, such as lasers, may reintroduce the break in symmetry even in the presence of very large photon numbers. Finally, we discuss the effects which arise from the consideration of finite numbers of interacting elements in the presence of a transition and draw a parallel with “small” systems from everyday’s experience.

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I. GENERAL REMARKS

Symmetries are one of the deepest properties which characterize the state of most real systems. In general, they are related to the invariance of some quantity. As a concrete example, we can consider the placement of objects in a space, let’s say furniture in a room, relative to its walls. When drawing plans, we can specify the position of each object starting from one chosen corner. However, as long as all positions are measured relative to the same point, the choice of corner is arbitrary, thus there is a symmetry in the problem (the room shape does not need to be symmetric, instead): any corner – or any other point of the room, such as its center – can be taken as reference. This property takes also the name of invariance: the result, i.e., the placement of the furniture in the room, does not change with the choice of reference point (as long as we don’t make a mistake in the drawings!).

The topic is extremely vast and is the object of deep investigations which involve mathematics, physics, chemistry, biology, etc. In this contribution we are going to focus onto some very specific examples to show how the asymmetry, as opposed to symmetry, is also intrinsic in nature and how it provides us with very interesting examples and applications. Some of them are all around us and part of everyday’s life, even though we might not notice them.

The paper is meant to be accessible to a broad audience, thus it will try to introduce concepts at a simple and heuristic level, without any pretension of rigour. The specialists will know how to fix the imprecisions of the presentation, necessary in order to avoid confusing the general reader with too many details. At the same time, we will try to arrive to some examples and applications of symmetries at a more advanced level. These parts may serve as an overview of some topics, presented in the fashion of a broad review that may be of some use also to more advanced Readers, who are therefore invited to skim the better known parts of this manuscript to arrive to those sections which will, hopefully, be of some interest to them.

The paper is organized as follows: we start by presenting the intrinsic properties of photons (section II), where we first consider just one photon at the time (section II A) to then extend the discussion to the superposition of two photons (section II C), ending up with large ensembles of photons (section II D). A few elementary consequences of the photon’s intrinsic symmetry break in the interaction with matter are simply highlighted in section II B. We then turn to the discussion of asymmetries in modern devices (section III), concentrating on the laser as an example (section III A). Questions arising from small sample sizes are analyzed in the context of small lasers (section III B) and their relation to collective behaviour in small human groups is briefly illustrated in section IV.

II. PHOTONS

Light is one of the most ubiquitous resources we live with: it allows us to have visual perception, but also enables the growth of organisms, the exchange between carbon dioxide and oxygen, etc. Light consists of very small, elementary and discrete entities called photons, or light quanta (discrete “light pieces”). This discretization is not exclusive to light, as the restriction of electromagnetic radiation to the visible (for us) part of the spectrum. All electromagnetic radiation, extending from radio-waves (long, medium and short waves, radar and microwaves) all the way to x-rays (e.g., used in medical imaging) and γ-rays (e.g., arriving from the Sun) – passing through the infrared and ultraviolet part of the spectrum – consists of discrete photons. An interesting chart illustrating the full electromagnetic spectrum and its properties can be found in Internet [1].

A. One photon at the time

Since we view the photon as an individual piece of light, the first question which comes to mind is: what are its features? Photons possess neither mass nor electrical
charge [2] but they carry energy (by which we distinguish colours) and another quantity which is particular interest for this discussion: spin. The best way of picturing spin is thinking of a toy top: at rest, it lays down on the surface on which it reposes, but when spun, it will turn either clockwise or counterclockwise (since we can choose how to make it rotate). Even though this is mostly an analogy, we can think of the photon as a spinning top which can turn in either direction: for definiteness, we will assign the value $+1$ to a photon spinning counterclockwise and $-1$ to one spinning clockwise.

In order to get another intuitive picture for the photon’s spin, we are going to think of another analogy: that of a string. The reason is that, as we mentioned earlier, photons are the *elementary pieces* of (electromagnetic) waves. Waves exist in many other media (e.g., acoustic waves in air) and in particular on strings. Thus, let us imagine holding a long piece of string attached at one end: one can either let it fall limp or pull it tight, but in neither case this produces a wave. An easy way of producing a nice wave is to rotate the free end of the string (the one in your hand) either clockwise or counterclockwise (cf., e.g., Fig. 1). By doing this fast enough, we see our string forming a spiral wave all the way down until the fastened end, just as a corkscrew (or a spiral staircase laying down). If the rotation is counterclockwise, following our previous convention, we assign it the value $+1$, otherwise $-1$.

We can further advance our understanding if we now look at the rotating string from its end (imagine having attached it to a transparent, glass, wall). If we just look at the motion of one point of the string, marked for instance by a coloured dot, we see it tracing a circle either with counter- or with clockwise motion (cf. Fig. 2). This is another way of picturing the photon’s spin: we assign to it a circular motion $+1$ when counter-, $-1$ when clockwise. To this motion we associate the concept of circular polarization, with the same signs used up until now.

This would restore a symmetric state between right and left. Various fundamental physical arguments can be given for the reason why such a state cannot exist (although in general any physical system with spin $\pm 1$ also has a 0 component), but this is left for specialists. A good overview of the topic, and some references in the literature, can be obtained from the web [3]. Here, we are going to use, again, an analogy to explain the reason why the “0 spin” state cannot exist for photons.

Going back to our example with the top, we can spin it either clockwise or counterclockwise. However, if we do not impart it any motion, the top is in an unstable state and topples over at the slightest perturbation: we cannot have it in a non-spinning state! We can therefore continue our analogy saying that we cannot have a photon with 0 spin. A different analogy is based again on the string: choosing either counter- or clockwise motion, we get a corkscrew kind of wave, but it is impossible to set up a wave that does not spin — a “counterexample” is going to be discussed in section II C, where we will clearly see that a non-rotating wave requires something more than a single photon.

FIG. 1: Illustration of a left-circularly (left sketch) and right-circularly (right sketch) polarized photon (i.e., wave) propagating in the direction marked by the arrow.

The existence of the two opposite states of polarization represents a spontaneous break of the intrinsic symmetry, since a photon can exist only with either handedness in polarization: left or right.

The question naturally arises whether there is the possibility for the existence of a photon with “0 spin” — this would restore a symmetric state between right and left. Various fundamental physical arguments can be given for the reason why such a state cannot exist (although in general any physical system with spin $\pm 1$ also has a 0 component), but this is left for specialists. A good overview of the topic, and some references in the literature, can be obtained from the web [3]. Here, we are going to use, again, an analogy to explain the reason why the “0 spin” state cannot exist for photons.

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FIG. 2: Illustration of a left-circularly (left sketch) and right-circularly (right sketch) polarized photon seen from front (head-on). The spiraling outer trajectories illustrate the perspective (i.e., taking into account the direction of propagation towards the Reader — i.e., out of the page), while the thicker circles at the respective figure centers eliminate the perspective, showing that the trajectory is entirely circular. The arrows on top of the thicker circles denote the direction of rotation.

B. Interaction between one photon and matter

Let us now ask a simple question: what happens when one photon encounters “matter”? Is there any interaction and what are its rules? Since we are focussing on the spin properties, we will particularly concentrate on this point, but first we need to settle the question of energy conservation: the interaction between the photon and matter will occur only if the exact energy of the photon can be exchanged back and forth. In order to keep the discussion as simple as possible, we consider an atom: its energy states are known to be discrete (think of the spectrum obtainable from various
metals: coloured flames in a chemistry lab, coloured fluorescent tubes – the old red neon tubes –, or the emission from a spectral lamp [4], etc.). We schematically represent these discrete states as the two horizontal lines of Fig. 3a, where the lower level has less energy than the upper state and the energy jump is equal to the photon’s energy (in mathematical terms: $E_2 - E_1 = \hbar \omega$, where $\hbar$ is a numerical constant – Planck’s constant – and $\omega$ is proportional to the frequency of oscillation of the “photon wave”). The possibility for a photon to be absorbed (or emitted) in the interaction with matter requires energy matching, which in the following discussion will be consider as automatically satisfied.

![FIG. 3: Illustration of atomic states (simplest kind of “matter”) for interaction with single photons. (a) Two discrete (or “quantized”) atomic states of energy $E_1 < E_2$ in the so-called “Kastler representation”. (b) The upper state of the atom possesses spin with components $\pm 1, 0$ (allowed for the atom), identified by the value of $m$. The double arrows show the possible interactions between circularly polarized photons and the atom. No interaction is possible between the state $(E_2, m = 0)$ and $(E_1, m = 0)$ through the exchange of a photon [5].]

Fig. 3b illustrates a somewhat more complex situation, where one atomic level (say the upper one with energy $E_2$) also possesses spin, denoted by the letter $m$. In the case of the atom, all three components are allowed ($m = \pm 1, 0$) and the interaction with the photon is subject to the restriction that the spin exchanged is conserved. Thus, the two possible paths are those indicated by the double arrows, either towards $m = +1$ or towards $m = -1$: the interaction between the two $m = 0$ states is not possible by exchanging a single photon, since the exchange requires a change in spin in the atomic state, while these two states have identical ($m = 0$) spin values (thus no “spin change” is involved in this transition). This is an important point which is at the basis of the questions posed on the chirality (i.e., handedness) of simple organic molecules as found in nature [6].

In other words, not all interactions are allowed and light selects those possibilities which are compatible with its broken symmetry, thereby imprinting the symmetry break onto matter.

C. Two photons

One photon does not carry a large amount of energy (for visible photons $E_{phot} \approx 2 \times 10^{-19} J$, i.e., $\approx 0.000000000005$ times the amount of energy needed to raise one drop of water by 1 mm). It therefore makes sense to see what happens if we take more than one photon simultaneously. The first step is to look at two photons. Of course, we are going to discuss the combination of two photons possessing the same energy (otherwise we simply get two “colours”).

If the photons have the same handedness, we simply get twice the energy with the same spin, but if the two photons have opposite handedness we get the result of Fig. 4. On the left, we combine two photons starting from a particular “initial position” (i.e., technically “phase” = 0°): the two dots represent the motion of the two photons on the circle of Fig. 2 (inner circle) rotating in opposite directions (arrows). The composition of the two rotating trajectories gives a trajectory which oscillates in the diagonal plane (tilted to the right), representing the actual motion of the wave. In the string analogy, there is no longer a helical wave, but an oscillation of the string in the diagonal plane drawn in the figure. This is of course possible, as one can easily test with a piece of rope (it may be difficult to get the direction right, but possible).

![FIG. 4: Combination of two photons with opposite handedness. The left panel illustrates the rotation of the two photons (in opposite directions) starting from the horizontal axis: the resulting polarization is linear along the diagonal of the first and third quadrant. The right panel shows that a different choice of “starting point” (on the vertical, one point up, the other down) gives rise to a linear polarization along the other diagonal. As explained in the text, all possible linear directions can be obtained with suitable transformations.

Simply starting from a different initial position (phase) it is possible to obtain a different oscillation plane (right sketch in Fig. 4). When testing the analogy with the rope, one is not simultaneously rotating one’s hand left and right simultaneously, but moving it diagonally up and down to generate the corresponding wave. The practical illustration tells us immediately – as can be done with mathematical transformations which we leave for specialized textbooks [7] – that there is an infinite number of planes along which one can make the string oscillate. Thus, there is also an infinite number of planes along which the oscillation of the radiation resulting from the combination of two photons with opposite handedness!

These considerations highlight several points:

a. The combination of the two photons with opposite handedness results in a state which has lost the apparent symmetry break;
b. The spatial symmetry is, however, lost because there is a preferential plane of oscillation;

c. The number of planes is infinite, thus this symmetry break is not a fundamental one, unlike the one related to the handedness, which rests on only two states.

The polarization states illustrated in Fig. 4 correspond to what is normally known a linear polarization. The fact that we need two photons with opposite handedness to obtain this state proves that linearly polarized photons do not exist, but that we can obtain linearly polarized light only by superposition of suitably paired photons (even in large number to obtain intense light, as long as each circularly polarized photon has its counterpart). Since the interaction with matter occurs one photon at the time (unless nonlinear, very high-power interactions are considered), linearly polarized light plays no role in transferring a symmetry break onto matter.

D. Unpolarized photons?

There is no such thing as an unpolarized photon. Unpolarized light, as received for instance from the sun, is the result of the superposition of pairs of photons, each producing a linearly polarized emission but without preference for the orientation of the plane of polarization.

Sticking to the sun as example, Earth receives approximately $10^{21}$ (i.e., a 1 followed by 21 zeros) photons per square meter per second (over the whole visible spectrum). Given this extremely large number of photons, it is easy to see how the random orientation of the plane of polarization (i.e., the axis in Fig. 4) produces light with no polarization preference. However, there is no such thing as an unpolarized photon, as there is no unpolarized wave – try making a string oscillate without any direction of oscillation, neither circular nor linear!

Thus, it is important to remark upon the fact that the only, truly symmetric state – a photon without polarization – can never exist!

III. ASYMMETRIES IN CURRENT OPTICAL APPLICATIONS AND DEVICES

The fundamental break in the symmetry related to the photon properties reflects in numerous applications. Their presence is at sometimes neutral or even beneficial, while in some cases a large amount of effort is placed into the reestablishment of a symmetric state for particular applications.

A. Lasers

One of the most ubiquitous devices which has entered everyone’s life is the laser. Discovered 58 years ago, and considered “a solution in search for a problem” [8], it has now found an innumerable number of applications, from telecommunications, medical, sensing, entertainment, construction, etc. Everyone has nowadays manipulated a laser at least as a pointing device or in a computer mouse.

The laser (Light Amplification by Stimulated Emission of Radiation) converts the spontaneously emitted photons, as those normally emitted by thermal sources, into a collimated beam of monochromatic (i.e., single frequency: one “colour”) radiation possessing a single “phase reference” (for an intuitive picture, think of a long wave which continues without breaks). As such, it has to perform some kind of selection on the kind of photons that it harbors inside and emits (in part). This is achieved by selectively exciting atoms into one single energy level (the upper one in Fig. 3), so that the emerging photons all possess the same frequency (or wavelength), then recycling the photons – i.e., bouncing them back and forth – with the help of two facing mirrors (forming an optical cavity). The latter strengthens the so-called stimulated emission process, whose existence was first postulated by Einstein in 1917 [9], responsible for the establishment of laser action.

Stimulated emission is a strongly nonlinear process – i.e., in everyday’s words, its outcome is not simply proportional to the ‘stimulus” but overreacts to it – which performs a selection in the photon’s properties, since it amplifies any instantaneous imbalance between the two photon spin states. We have previously seen that when the number of photons is large, there is, statistically, an equal number in each polarization state, left or right. The statistical balance, however, exists only when considering averages (over time or over different repetitions), exactly in the same way as we can predict a 50% outcome for head and tails when tossing a coin (alternately, between odd and even numbers). However, at the level of an actual sequence of tosses, we may find a sequence of multiple “heads” (say three or four) before getting a “tail”. Thus, at the “microscopic” scale we always have imbalances.

Even in a weak, barely visible laser the number of photons emitted in a second exceeds $10^{12}$ and, on the short timescales on which the laser field builds up, we typically find more than 1000 photons. While the imbalance may be small (think of a “head” and “tails” experiment), there is bound to be one. Because of the “reactiveness” of the amplification by stimulated emission to any excess, it is natural to expect that the photon class (say with “+1” polarization) which has a larger number of photons will win the competition and the laser will emit exclusively +1 photons. The intrinsic symmetry break is now translated into a macroscopic property, unlike what happens in the emission of large sources (e.g., the sun), thanks to the selectivity of stimulated emission and the overall lasing conditions.

We therefore dispose of a mechanism which transfers the intrinsic symmetry break of photons onto the
macroscopic state of light, providing an optical source whose characteristics strongly differ from those of any other thermal source (lampbulbs, sun or even spectral lamps). The previous considerations would imply that the actual state of polarization of the light would depend on the configuration (predominance of +1 or -1 polarized photons) at the moment the laser is turned on. This would indeed be the case if the technological implementation of the system (first of all the mirrors, but also any other element entering into the construction) were perfectly isotropic. This is, in general, not the case and the smallest anisotropy helps breaking the symmetry and imparting a preferential polarization on the device. In this case, the polarization will always be the same, thus rendering the use of the device more practical. Indeed, having a reproducible control on the characteristics of the emitted photons (in this case, their polarization) helps designing the properties of the optics which follows.

As an example, think of polarizing sunglasses: their principle of operation is to remove one of the two polarized components out of the sunlight. Given the “unpolarized” state of sunlight (equivalent number of photons emitted in all planes of polarization) this would remove half of the light which arrives to the eye. However, reflection from surfaces tends to privilege one polarization component over the other [7] thus aligning the axis of the polarization filter (i.e., the “sunglass”) to remove the reflected components attenuates the light even further, lending comfort to the eye. Those who have used polarizing sunglasses may, however, have noticed that it is impossible to read at certain angles digital, liquid-crystal-based displays, such as those of wristwatches. These displays are also polarized, because of construction needs, and if your wrist happens to be turned at the wrong angle, the light arriving from your wristwatch will be entirely filtered: in such a case, the display appears to be completely dark [10].

While the control of the broken polarization state is useful for practical purposes, there are interesting applications for the balanced situation, where equal number of photons in each polarization state are emitted by a laser. In this case, the macroscopic equilibrium is reestablished, not through “unpolarized photons”, but through the balance in the number of photons of each kind. This is an extremely difficult condition to satisfy, due to the sensitivity of the nonlinear amplification process described above, given its intrinsic lack of stability. One can picture the difficulty in achieving this condition with the following analogy: you can hold a long rod on the tip of your finger, as long as it stays perfectly vertical. Since external influences will perturb it away from that position, you need to continuously compensate by moving your finger to keep balance. The same is needed in an “unpolarized” laser and requires serious efforts. Such efforts are, however, repaid by the sensitivity of the resulting device to external perturbations. One example for possible uses of the “unpolarized” laser (or “polarization isotropic laser”) is its sensitivity in monitoring tiny magnetic fields [14]. More recently, applications of isotropic lasers for detection in biophotonics have been envisaged [15].

B. Small lasers

In the last couple of decades there has been keen technological interest in considerably reducing the volume of the laser cavity. The biggest push behind this interest comes from a considerable reduction in energy required to supply a tiny laser, as compared to a macroscopic one, accompanied by lower thermal dissipation and a diminished footprint. The main scope of these investigations is to reach devices so small as to fit into a chip the size of an electronic one, with the ultimate aim of replacing electronics with light. Indeed, the speed (of light) at which the information can flow in such a chip is about two orders of magnitude larger than the electron speed in a circuit. Small volumes are necessary to pack a large number of components – to be competitive with electronics – while low consumption and low thermal dissipation are required for compatibility with dense packing. Such chips are now already being built [16], but the light sources (lasers) are still external and feed the optical circuit through fibers.

The reduction in cavity volume, however, carries with it an additional problem related to the small number of “elements” [17] participating in the laser emission. This deviation from large number laws, which characterize the preceding discussions, introduce an additional element of randomness which provoke more complex transitions between states. We are going to touch on a couple of problems and will conclude, in the next section, with an analogy in other systems which can be interpreted with the help of physics and optics.

We first remain within the framework of the polarization states of the photon and look at the transition between random light (like a Light Emitting Device – LED) and lasing in a semiconductor laser (specifically a VCSEL: Vertical Cavity Surface Emitting Laser). When the cavity volume is very small, the discrimination between the states weakens and it is possible to obtain a much more complex behaviour, where the second, weaker polarization component also starts to emit, but then is suppressed when the stronger one takes over. Due to manufacturing-induced defects it is, in addition, possible to observe a switch in the polarization properties during operation with simple bistability (i.e., the stable emission of either polarization) or with dynamics (alternation between the two states) [18]. In our group, we are in the process of investigating these questions in microlasers in order to understand, on the one hand, the physics of the interactions and, on the other hand, the issues at stake related to finite size effects (cf. below).

As the cavity size is reduced, another form of
competition becomes apparent, which illustrates a different aspect – unrelated to the polarization states of photons. Since the collective interaction which induces the transition towards lasing relies on the existence of a large number of elements [17], their reduction, due to the small cavity size, opens questions connected to small numbers (finite size effects). It has been known for nearly 30 years that the transition between the two kinds of behaviour (LED-like light and laser light) is not as well-defined [19, 20] with questions which have persisted until recently [21, 22].

We have been able to show, in the past couple of years [23, 24], that the transition between the two states goes through a sequence of intermediate steps [25] which can be recognized in other systems, as discussed in the following section.

IV. AN INTERESTING ANALOGY

The transition between states in the presence of a small number of “elements” (in a generic sense here) is something which is part of everyday’s experience, even though it is not necessarily recognized as such. For lack of space, I am going to limit this discussion to one single example which is known to everyone: audience clapping at a performance.

It is well-known and recognized that depending on the degree of enthusiasm, a group of people will clap in a more or less energetic way. What is less immediately recognized, however, is the fact that different clapping regimes exist, ranging from desultory and weak clapping to the so-called standing ovation and rhythmic clapping [26–28]. The cited investigations have studied the transition from disorderly clapping to rhythmic applause in small test groups, but it is known from experience that only large enough audiences will spontaneously make the transition to a regular applause [29].

The analogy with the transition between LED-like and laser light can serve as good guidance, as there appears to be interesting similarities in the behaviour [25], which we cannot detail here, with the added benefit that the laser is not subject to external influences such as psychology, group pressure, external signals and the like.

V. CONCLUSIONS

We have presented in a qualitative way the fundamental symmetry break intrinsic to the nature of light and shown its implications into the various forms under which we either receive and perceive radiation and on its interactions with matter. The intrinsic symmetry break disappears when we look at large ensembles, such as the light which we receive from the sun. However, current technological devices (such as lasers) through their peculiar interactions transfer the microscopic photon properties to the macroscopic scale. Finally, we have shown that when considering systems of reduced size, the transitions become less well-defined and have given an example from everyday’s life showing how it is possible to draw interesting parallels between optical/physical systems and the dynamics of small groups, with potential for interpretations of the observations based on phenomena which are not influenced by human nature.

[2] Current measurements ensure that the mass of the photon, if it exists at all, is smaller than $10^{-53} \text{Kg}$, i.e., $0\ldots$ where the number of zeroes which follow the dot is 53. For comparison, the mass of an electron, an extremely light particle, is $10^{22}$ times larger than this upper limit (i.e., 10 followed by 21 zeros, i.e., one thousand billion billions). Similarly, the electrical charge of the photon, if it exists, is smaller than $10^{-35} \text{C}$, i.e., $10^{16}$ times smaller than the charge of the electron (the smallest known free charge). The reason why we do not say that mass and charge are 0 is that measurements are limited by the sensitivity of the apparatus and can never find a true 0, but can give only an upper limit (instrumental sensitivity) to “what we may be missing”. In other words, the instruments reads 0 within the limitation (error bar) of its sensitivity, which becomes the largest value (for mass or charge) that is compatible with the 0 reading.
[4] The emission spectra of various kinds of atoms (e.g. Sodium, Hydrogen, Calcium and Mercury) obtained from spectral lamps can be found at: http://www.didamoe.com/mercury-vapor-lamp-spectral-lines/ – consulted on June 12, 2018. Each colour corresponds to a different distance between the energy levels.
[5] These considerations hold when we keep the reference axis unique for all “elements” involved in the discussion (here photons and atoms). When this is not the case, it is possible to induce transitions between states with same value of $m$, but this issue is too complex for the general discussion offered here.


10. I used to be unable to read my digital wristwatch when driving with polarized sunglasses because of the orientation of my wrist on the steering wheel.


17. By “elements” we mean here emitters (e.g., atoms or equivalent elementary components in matter) and resonances in the cavity volume.


29. The content of this paper is based on material presented at the First European Asymmetry Symposium, held in Nice, France, on March 15–16, 2018. An interesting concert was given the evening of the first day, attended by just over 100 people (maybe as many as 140). There, we had an experimental demonstration of this fact: the audience, though enthusiastic about the performance, never quite reached the point of synchronized clapping, in spite of a tendency noticed by the author of this paper – who was attentively listening for a potential for switch in clapping style. Even trying to force the synchronization (by clapping harder with a group which seemed on the point of prevailing) did not suffice to make the transition.