# The Problem of Action at a Distance in Networks and the Emergence of Preferential Attachment from Triadic Closure

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**Abstract**

In this paper, we characterise the notion of preferential attachment in networks as action at a distance, and argue that it can only be an emergent phenomenon – the actual mechanism by which networks grow always being the closing of triangles. After a review of the concepts of triangle closing and preferential attachment, we present our argument, as well as a simplified model in which preferential attachment can be derived mathematically from triangle closing. Additionally, we perform experiments on synthetic graphs to demonstrate the emergence of preferential attachment in graph growth models based only on triangle closing.

**Keywords**

networks; preferential attachment; triangle closing; action at a distance

## I  INTRODUCTION

Many natural and man-made phenomena are networks – i.e., ensembles of interconnected entities. To understand such structures is to understand their creation, their evolution and their decay. In fact, many models have been proposed for the evolution of networks, for the simple reason that a very large number of real-world systems can be modelled as networks. Rules for the evolution of networks can be broadly classified into two classes: those postulating local growth, and those postulating global growth. An example for a mechanism of local growth is triangle closing: When two people become friends because they have a common friend, then a new triangle is formed, consisting of three persons.[1] This tendency of networks to form triang-

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[1] In this paper, we use the terms *triangle closing* and *triadic closure* exchangably. The notion of triadic closure

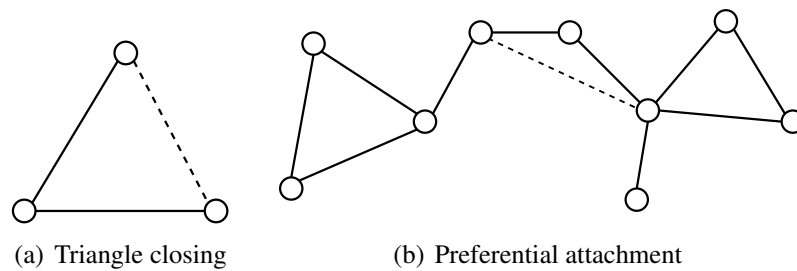(a) Triangle closing      (b) Preferential attachment

Figure 1: The two network growth mechanisms considered in this article: triangle closing and preferential attachment. In both models, new edges appear (shown as dashed lines), based on the network environment of the current graph. (a) Triangle closing: an edge is more likely to appear between nodes that have common neighbours, (b) Preferential attachment: A node with higher degree is more likely to receive an edge.

les is a natural model not only for social networks, but for almost all types of networked data. For instance, if Alice likes a movie and Bob is a friend of Alice, Bob might also come to like that movie. In this case, the triangle consists of two persons and one movie. In general, networks can contain any type of object being connected by many different types of connections, and thus many different types of such triangle closings are possible. We call this type of growth *local* because it only depends on the immediate neighbourhood of the two connected nodes; the rest of the network does not play a role.

In contrast to local graph growth rules, there is the phenomenon of preferential attachment. When, for instance, two people become friends with each other, not because they have a common friend, or go to the same class, but because one of them or both of them are popular.[2] Given a popular person, i.e. with many friends, it is more likely that he will be chosen as a friend, than an unpopular person, all else being equal. This phenomenon is referred to as preferential attachment. Preferential attachment is an often-used strategy to predict new connections, not only in social networks: a frequent movie-goer is much more likely to watch a popular film, than someone who almost never goes out to the movies watching an obscure film almost nobody knows or has seen. These types of statements seem obviously true and indeed they are used widely in application systems: recommender systems give a big preference to popular movies, search engines give higher weight to well-connected web pages, and Facebook or Twitter will make a point to show you pictures that already have many likes. In that sense, preferential attachment is true empirically, and has been verified many times in experiments. However, preferential attachment has one problematic property: It relies on connecting any two completely unrelated nodes, merely because of their degree, without considering their interconnections. Preferential attachment can thus be labelled as "action at a distance". For this reason, we argue that preferential attachment is never a primitive phenomenon, but always a derived phenomenon, emerging as a result of more basic network evolution rules, which themselves do not involve action at a distance.

So, if preferential attachment is not a primitive network evolution mechanism, which network evolution rules should then be considered as primitive in our network growth model? We will present in this paper arguments for the thesis that only the principle of triangle closing is fun-

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has been alluded to multiple times in the history of the social sciences; and became mainstream with the work of Mark Granovetter (1985).

   [2]Preferential attachment, too, is a concept with a long history, having been alluded to under multiple names. See the references in (Kunegis *et al.*, 2013) for an account of the early work on it.

damental, all forms of preferential attachment being derived from it. To give an argument in favour of our thesis, we will first review basic notions of networks and network evolution models, and then review preferential attachment, proposing various mechanisms by which it can arise from triangle closing, a fundamental notion in the evolution of networks. Finally, we perform experiments on synthetic graphs to test to what extent preferential attachment may emerge from graph growth models that include only triangle closing and/or random addition of edges.

## II    RELATED WORK

The debate over the nature of preferential attachment mechanisms dates back to the 1960s, when the economist Herbert Simon defended the role of randomness and the mathematician Benoît Mandelbrot defended the role of optimisation (Barabási, 2012). The concept of preferential attachment is also used to explain the nature of scale-free degree distributions in biological networks such as metabolic networks (Jeong *et al.*, 2000) and protein networks (Jeong *et al.*, 2001). There are various suggestions to explain the nature of preferential attachment for instance by introducing hidden variable models in which nodes possess an intrinsic fitness to other nodes in unipartite (Boguñá and Pastor-Satorras, 2003) or bipartite networks (Kitsak and Krioukov, 2011). In a recent *Nature* paper, Papadopoulos and colleagues proposed a model based on geometric optimisation of homophily space (2012). However, in these models, triadic closure is not defined as the main principle for the formation of edges.

Triadic closure, a tendency to connect to the friend of a friend (Rapoport, 1953), has been observed undeniably in many social networks such as friendship at a university (Kossinets and Watts, 2006), scientific collaborations (Newman, 2001) and in the World Wide Web (Adamic, 1999). The concept of triadic closure was first suggested by German sociologist Georg Simmel and colleagues (1950) and later on popularised by Fritz Heider and Mark Granovetter as the theory of cognitive balance in which if two individuals feel the same way about an object or a person, they seek closure by closing the triad between themselves (Heider, 2013). Since the classic preferential attachment model fails to explain the number of clusters in many social networks, many attempts have been made to include triadic closure to the model (Holme and Kim, 2002; Vázquez, 2003), in which nodes connect with certain probabilities based on the principle of triadic closure. These works have shown that the scaling law for the degree distribution and clustering coefficient can be reproduced based on these models (Klimek and Thurner, 2013). Similarly, models based on random walks as local processes have been proposed, too, of which triangle closing is a special case (Evans and Saramäki, 2005).

Hence, the scale-free nature of networks and the abundance of triangles in socio-technical networks beg for a more fundamental explanation. Moreover, the observable part of these systems is not necessarily completely representative for the entire system. Networks are generally multi-layered or multiplex, in which some layers can be hidden or simply not possible to observe (Kivelä *et al.*, 2014). For instance, the creation of a new Facebook tie can be caused by attending the same class, sharing the same hobby or living in a same neighbourhood, which is hidden from the observable data. Consequently, these *focal* points contribute to the tie creation known as *focal closure* and need to be considered in modelling realistic networks, as argued by Kossinets and Watts (2006).

## III    NETWORKS

The assertion that networks are to be found everywhere has become a cliché because it is true. Social networks, knowledge networks, information networks, communication networks – many papers in the field of network science motivate their use by enumerating fields in which they

play a central role. Biological networks, molecules, lexical networks, Feynman diagrams – hardly a scientific field exists in which networks do not play a fundamental role. Instead of giving a hopelessly incomplete enumeration of examples, we will simply refer the reader to the introductory section of our Handbook of Network Analysis (Kunegis, 2017), in case she wishes to convince herself of this fact. In case this is not enough, we may point to the existence of entire fields of research incorporating the word *network* and synonyms that have emerged in the last decade: network science (Börner *et al.*, 2007; Newman, 2010), web science (Hendler *et al.*, 2008), and others (Tiropanis *et al.*, 2015). There are many ways to justify the ubiquitous use of networks as a model. As an example, we may consider their use in the field of machine learning. Most classical machine learning algorithms deal with datasets consisting of data points, each consisting of the same features. Mathematically, we may model such a dataset as a set of points in a space whose dimensions are the individual features (Salton *et al.*, 1975). This formalism is very powerful, and still constitutes the backbone of many machine learning and data mining methods to this day. The standard formulation of classification, clustering and other learning problems all rely on the set-of-points-in-a-space model. However, not all machine learning problems are well described by the *set of points* model. While the set of words contained in text documents are well represented by the *bag of words* model (Baeza-Yates and Ribeiro-Neto, 1999), a social network is not. We may try to represent a social network as a bag of friends, but this representation is very unsatisfactory: each person has a set of friends, but the model does not reflect the fact that a person contained in one of these bags is the same person as one *having* a bag of friends. Thus, the vector space model cannot find connections such as "the friend of my friend" – it can only find "a person that has the same friend as me". In other words, the vector space model disconnects the role of *having friends* and that of *being a friend*. Instead, the natural way to represent friendships is as a network. Using a network model, the symmetry of the friend relationship is included automatically in the model, and relationships such as *the friend of my friend* arise as the natural way to create new edges in the network, i.e., triangle closing. In fact, we will argue that this is the only way new edges can be created in a network, and that other models are merely consequences of it, such as preferential attachment.

As an additional remark, the terms *network* and *graph* are often used interchangeably. Strictly speaking, a network is the real-world object to be analysed, such as a social network, while a graph is a mathematical structure used to model it.

## IV PREFERENTIAL ATTACHMENT
Preferential attachment, also referred to by the phrase "the rich get richer", or as the Matthew effect, is observed empirically in many social networks (Kunegis *et al.*, 2013). In fact, the phenomenon of preferential attachment is known by many other names in different contexts; see the references within (Kunegis *et al.*, 2013) for an account. In other words, who has many friends, will get more new friends than who has few. Movies that have been seen by many people will be seen by more people than movies that have not. Websites that have been linked to many times will receive more new links because of this. These statements seem true, and indeed, they are true empirically for many different network types.

In fact, preferential attachment is the basis for a whole class of network models. The most basic of these, the model of Barabási and Albert (1999), describes the growth of a network, which proceeds as follow: Start with a small graph, and at each step, add a node, and connect that node to $k$ existing nodes with a probability proportional to the number of neighbours for each existing node. In the limit where many nodes have been added in that way, the network tends to become *scale-free*, i.e. tends to have a distribution of neighbour counts that follow a power

law. Since power law degree distributions are observed in many natural networks, the usual conclusion is that preferential attachment is correct.

Preferential attachment is thus undeniably real. Why then, are we arguing against it? The reason is that preferential attachment cannot be a fundamental driving force for tie creation. How are two nodes, completely unconnected from each other, be supposed to choose to connect with each other? How can two completely disconnected nodes even *know* of each other's existence? This is a fundamental problem with all nonlocal interactions. For instance, the classical theory of gravitation as defined and used by Isaac Newton (1687) includes nonlocal interactions. In that theory, two masses exert a force on each other, regardless of their position. While the force decreases with distance, it is always nonzero, and instantaneous. The conceptual problem with this type of interaction had been identified already by Newton himself (Hesse, 1955). In modern physics, Newton's formalism is replaced by more precise theories that do not include any action at a distance. The theory of general relativity as defined by Albert Einstein (1916) for instance, only includes local interactions in the form of the Einstein field equations. Einstein's general relativity is thus free from any problematic *action at a distance*, and has been verified at many experimental scales. This is also true for other types of physical interactions – instead of a force that acts at a distance between matter particles, quantum field theory models *bosons* that connect particles. In fact, such interactions can be represented by Feynman diagrams: graph-like representations of particles in which edges are particles and nodes are interactions – any interacting particles must be connected in one diagram, directly or indirectly. In this light, we may interpret preferential attachment as a theory that is true superficially, but must be explained by an underlying phenomenon. Specifically, an underlying phenomenon that does not rely on action at a distance. As this phenomenon, we propose the known mechanism of triangle closing.

## V  TRIANGLE CLOSING

How do we make new friends? By meeting the friends of our friends. This represents a triangle formed by ourselves, our previous friend, and our new friend. What if we meet our new friend in another way – maybe at a party, or a concert, or at work ... in any case, there is always *some* element in common. If we meet our new friend at a party, then we are both connected to the party, and by modelling the party as a node in our network, that new friendship is indeed created by the closing of a person–person–party triangle. Of course, we may continue to ask how our connection to the party arose. After all, we did not come to a party randomly. No, we came to the party because a friend invited us, or for any other reason, as long as there is some connection. This game of connections can be played to any desired degree of precision. Maybe we *really* went from door to door until we found a party with many people. But then, how did we get from door to door? We surely must have started somewhere, likely near to our home, and have then gone on to the next door, and to the next door, and so on. In doing this, we have only followed links: We are connected to our home by living there; our home is connected to the neighbouring house, which itself is connected to the next house, and so on. This example is of course exaggerated, but serves to illustrate the principle: in order for a new edge to appear, a path has to exist from one node to another; this can go over nodes representing any type of entity, and these nodes may be visible or hidden. All in all, there is no escaping the principle of triangle closing. However we arrived at the party, it must have been by a series of triangle closings.

Thus, triangle closing fulfils the expected role as a fundamental mechanism of network growth, as it is purely local. However, we cannot deny the existence of preferential attachment, for which we must now find suitable explanations.

# VI  EXPLANATIONS

In recommender systems, such as those used on web sites that recommend movies to watch, preferential attachment is often taken as a solution to the cold start problem. The cold start problem in recommender systems refers to the situation in which a user has not yet entered any information about herself, and thus triangle closing cannot be used to recommend her anything. If the user has watched only a single movie, then we can find similar movies and recommend them. If a user has added only a single friend, then we can take movies liked by that friend and recommend them. But if the user is completely new, as has no friends and no ratings yet, then this strategy will not work. How then, do recommender systems give recommendations to new users? The solution is simple: they recommend the most popular items. If you subscribe to Twitter, you will be recommended popular accounts to follow. If you subscribe to Last.fm, you will be recommended popular music. For these sites, this strategy is better than not recommending anything, and in fact is a form of preferential attachment: Create, or rather recommend, links to nodes with many neighbours. How can we interpret this in terms of triangle closing? If a node has no connections yet, then surely it cannot acquire new nodes by triangle closing. How then will a node ever acquire new edges, if it starts without neighbours? The answer is that a node does not start without any neighbours. Everything is connected. A child when it is born does not start without connections; it is already connected to its parents and to its birthplace. Likewise, a user on the Web never starts from scratch: every page has a referrer, and thus the user can be connected to another website. Even if the referring web page is not known, there has to be a referrer. If a user types in a URL by hand, she has to have taken it somewhere: maybe a friend gave it to her, maybe she read it in a magazine, on a billboard, or on a truck . . . in all cases, the newly created connection is not created *ex nihilo* – it is created by triangle closing.

The explanation for preferential attachment thus lies in hidden nodes: Nodes that make indirect connections between things, but do not appear in the model. On Facebook for instance, many new friendships are created between people who do not have common friends. These new friendships seemingly appear without the help of triangle closing. However, that is always due to the fact that Facebook does not know everything. Some people are simply not on Facebook, which means that if one meets a new friend through a friend that is not on Facebook and then connects the new friend via Facebook, then from the point of view of Facebook a new edge was created without triangle closing. But that is only true because Facebook does not know my initial friend. If it did, it could correctly infer the new friendship via triangle closing. Thus, any two nodes in a network can potentially be linked, even if they do not share common neighbours *in the network at hand*, because they may share a hidden common neighbour. The same argumentation applies to hidden nodes that represent non-actors, such as classes, hometowns, parties, etc.

In order to justify preferential attachment as an emergent phenomenon, we must thus derive the mechanism that leads to edges being created specifically between nodes of high degrees. Consider a network, for instance a social network. Call this the known network. Then, consider a certain number of nodes outside of that network, that are connected at random to the nodes in the known network. Call these the unknown nodes. How many common neighbours do two members of the known network have outside of the known network? Without knowing the distribution of hidden edges, this question cannot be answered. But consider that triangle closing acts not only on known–unknown–known paths, but also on known–known–unknown paths. Starting with an equal probability for all known–unknown edges, performing triangle closing will lead to the creation of known–known–unknown triangles. The newly created known–unknown

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edges can then be combined with other unknown–known edges to perform, again, triangle closing, leading to new known–known edges. The result are new edges in the observed social network, with a probability proportional to the number of the initial known node's neighbours. Thus, preferential attachment emerges as a necessary consequence of iterated triangle closing, if hidden nodes are admitted. The next section will make this heuristic argument precise.

## VII DERIVATION

This section gives an exemplary derivation of a simplified model that we introduce to illustrate that preferential attachment arises as a consequence of triangle closing in the presence of hidden nodes. The given scenario is very general and may be generalised easily for instance by considering multiple node types or multiple edge types. In this model, we distinguish two types of nodes: visible nodes in the set $V$, and hidden nodes in the set $W$. We will assume that there is a given, fixed number of visible nodes $|V|$, and a possibly very large number of hidden nodes $|W|$. In particular, we will consider the limit $|W| \to \infty$.

Let $G = (V \cup W, E)$ be the graph representing the complete system, in which $V$ is the set of visible nodes, and $W$ the set of hidden nodes. Additionally, $E$ is the set of edges connecting nodes in $V$ with nodes in $W$. While we assume that the individual edges in $E$ are hidden, the degree of the nodes in $V$ is not hidden. In other words, the number of edges of $E$ incident to each node in $V$ is known. Edges between nodes in $V$ will not be considered. Likewise, edges between nodes in $W$ need not be considered, since they do not contribute to the degree of nodes in $V$. Thus, the considered network $G$ is bipartite. We will use the convention that $n = |W|$, and the degree of a node $u$ is denoted by $d(u)$. We now assume that the graph $G$ will receive new edges according to the principle of triangle closing. Thus, two nodes in $V$ will connect with a probability proportional to the number of common neighbours they have. Seeing only nodes in $V$ and their degree, preferential attachment can then be observed as described in the following.

In order to make our derivation, we need to make two assumptions:
- The triangle closing process is random in the sense that new edges are added between any possible node pairs with equal probability.
- The typical degree of nodes is significantly smaller than the number of nodes, i.e., $d(x) \ll n$. This is precise when $n$ goes to infinity.

Let $u, v \in V$ be two nodes of the network. Under the assumption that the edges are distributed randomly in the graph, the probability $p$ that $u$ and $v$ are connected can be derived combinatorically by considering the number of configurations in which the two nodes do not share a common neighbour. Given that $u$ and $v$ have degree $d(u)$ and $d(v)$ respectively, the total number of configurations for the edges connected to the nodes is

$$\binom{n}{d(u)}\binom{n}{d(v)}. \tag{1}$$

Out of those, the number of configurations in which the neighbours of the two nodes are disjoint is given by

$$\binom{n}{d(u)}\binom{n - d(u)}{d(v)}. \tag{2}$$

Thus, the probability that the two nodes share a common neighbour is given by

$$p = 1 - \frac{\binom{n}{d(u)}\binom{n-d(u)}{d(v)}}{\binom{n}{d(u)}\binom{n}{d(v)}} = 1 - \frac{\binom{n-d(u)}{d(v)}}{\binom{n}{d(v)}}. \tag{3}$$

We now use the falling factorial to express binomial coefficients, i.e.,

$$n^{\underline{a}} = n(n-1)(n-2)\cdots(n-a+1). \tag{4}$$

The falling factorial has the property that in the limit where $a$ is constant and $n$ goes to infinity, we have

$$\lim_{n\to\infty} \frac{n^{\underline{a}}}{n^a} = 1 \tag{5}$$

and also,

$$\binom{n}{a} = \frac{n^{\underline{a}}}{a!}, \tag{6}$$

and thus

$$p = 1 - \frac{(n-d(u))^{\underline{d(v)}}d(v)!}{d(v)!n^{\underline{d(v)}}} = 1 - \frac{(n-d(u))^{\underline{d(v)}}}{n^{\underline{d(v)}}}. \tag{7}$$

In the limit when $n$ goes to infinity we may thus assume that

$$p = 1 - \frac{(n-d(u))^{d(v)}}{n^{d(v)}} = 1 - \left(1 - \frac{d(u)}{n}\right)^{d(v)} \tag{8}$$

and using again the limit $n \to \infty$, and the property that in the limit where $\varepsilon$ goes to zero, $(1-\varepsilon)^k$ goes to $(1 - k\varepsilon)$,

$$p = \frac{d(u)d(v)}{n}. \tag{9}$$

It thus follows that $p \sim d(u)d(v)$, i.e., the probability of the nodes $u$ and $v$ being connected is proportional to both $d(u)$ and $d(v)$. Thus, we find that preferential attachment is a consequence of the triangle closing model. Preferential attachment itself then leads to a scale-free degree distribution, as per Barabási and Albert (1999).

## VIII EXPERIMENTS

In this section, we give empirical evidence for the emergence of preferential attachment in graph growth models that do not include it. In the experiments, we generate synthetic networks via a random growth process that does not include preferential attachment, as well as using random growth processes that do include preferential attachment. In all generated networks, effects of preferential attachment are then measured empirically. All generated networks have 1,000 nodes and 10,000 edges, and are undirected, loopless, and do not allow multiple edges. In all cases, the graphs are generated by starting with a graph of 1,000 nodes and without edges, and adding edges one by one. For each edge that is added, one of the following three methods is chosen at random:

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- **Random**: With probability $p_{\mathrm{r}}$, an edge is added randomly between two unconnected nodes. All pairs of distinct unconnected nodes are chosen with equal probability.
- **Triangle closing**: With probability $p_{\mathrm{tc}}$, among all unclosed triads, one is chosen randomly with equal probability, and the third edge is added. An unclosed triad is a triple of nodes $(u, v, w)$ such that $(u, v)$ and $(u, w)$ are connected, but $v$ and $w$ are not connected. If chosen, the triangle is completed by adding the edge $(v, w)$. If no unclosed triads are present, an edge is added at random as described in the previous case.
- **Preferential attachment**: With probability $p_{\mathrm{pa}}$, a node is chosen with a probability proportional to the node's current degree. Then, out of all nodes not connected to that node, one is chosen randomly and with equal probability, and an edge is added between the two selected nodes. If there is not at least one unconnected pair of nodes with nonzero degree, an edge is added at random as described in the first case.

In each experimental trial, the three probabilities are chosen such that $p_{\mathrm{r}} + p_{\mathrm{tc}} + p_{\mathrm{pa}} = 1$. Each of these probabilities is varied from 0 to 1 in increments of 1/11, excluding the case $p_{\mathrm{r}} = 0$ in order to avoid the runaway case of an individual node accumulating all edges.[3]

First, in order to verify whether a graph created by the process of triangle closing display scale-free behaviour, we compare the generated distribution of the triangle closing case with the degree distributions for the random and preferential attachment cases.[4] All three degree distributions are shown in Figure 2. In the plot, several observations can be made. The degree distribution for the triangle closing case displays power law-like behaviour over multiple orders of magnitude, from the smallest degrees of one, to approximately one hundred. While the networks generated by triangle closing and preferential attachment have similar power-like degree distribution, both with comparable exponent, we must note that the maximum degree in the preferential attachment case is larger than in the triangle closing case. However, the triangle closing model displays a power law degree distribution with exponential cut-off that has been observed in many real networks due to finite size effect (Boguñá *et al.*, 2004; Clauset *et al.*, 2009). For comparison, the preferential attachment case also displays power law-like behaviour, although not for very small degrees (under about 10), and additionally has a well-defined long tail. The purely random case leads to a degree distribution that shows no scale-free behaviour.

We measure the equality of the distribution of edges, or its opposite, its skewness, as the primary consequence of the preferential attachment process. As a measure, we use the Gini coefficient of the degree distribution, as defined in (Kunegis and Preusse, 2012). The Gini coefficient is zero when all nodes have equal degree, and attains its theoretical maximum of one when all nodes except a single one have degree zero.[5]

The experimental results are shown in Figure 3. In the triangle shown in the figure, the top-to-bottom-right edge shows the cases in which preferential attachment is excluded, while the bottom-left corner (*Pref. att. = 100%*) represents the case of exclusive preferential attachment. As expected, the 100% random case results in an Erdős–Rényi graph in which the degrees have a Poisson distribution, and thus a very uniform number of edges over all nodes, giving a small Gini coefficient of 17.7%. The pure preferential attachment case gives a higher value

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[3]In the degenerate case of $p_{\mathrm{r}} = 0$, almost all edges will be attached to a single node in the $n \to \infty$ limit.

[4]As described in the previous paragraph, the cases of pure triangle closing and preferential attachment also include 1/11 of edges based on random assignment.

[5]Since an edge always connects two nodes, the actual maximum is attained in star graphs, in which all edges attach to a single node, and other nodes have a degree of zero and one. In the large-graph limit, the Gini coefficient in such graphs tends to one.

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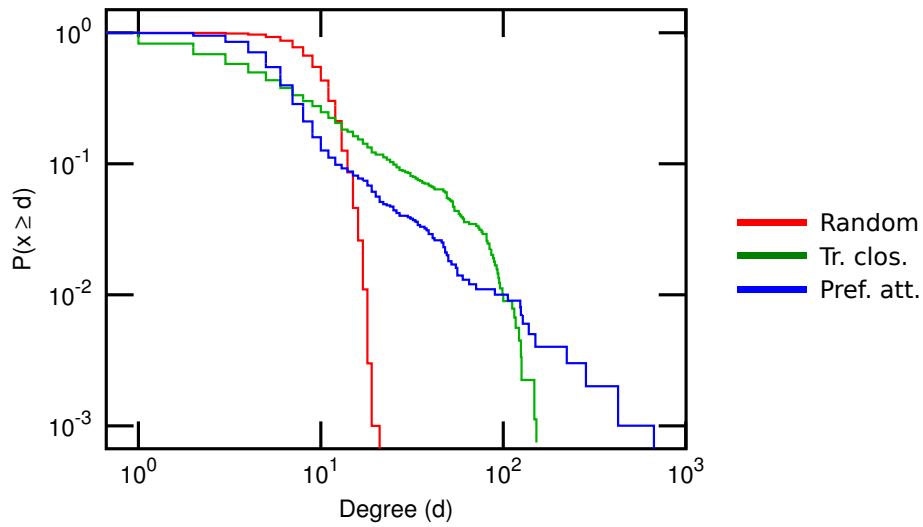Volume: **2** - Year: **2017** , *DOI: 10.18713/JIMIS-140417-2-4*

Figure 2: The cumulative degree distribution for the three extremal generated networks.
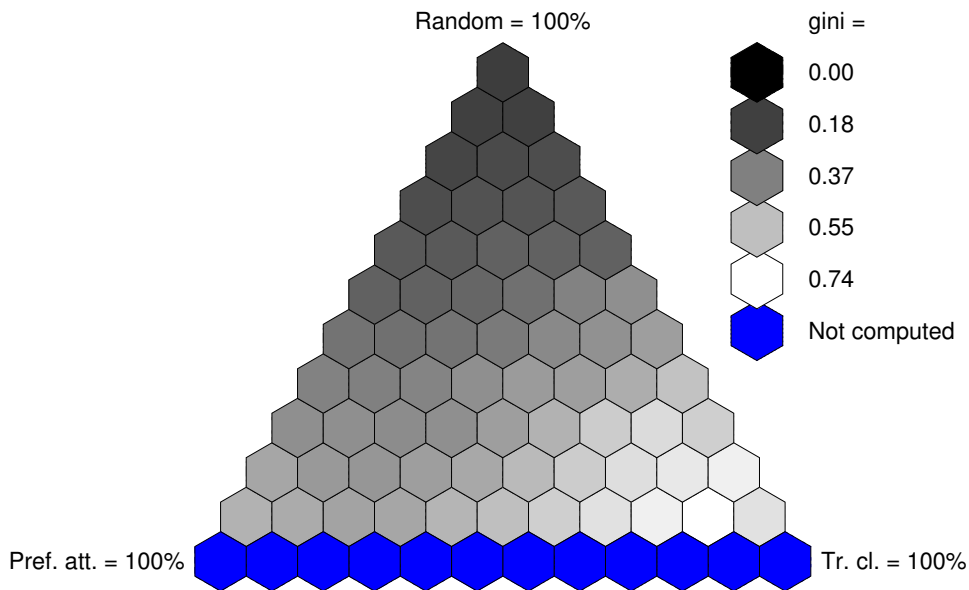


Figure 3: Experimental results: Each cell shows one experimental run with a different probability of adding each edge at random (top), via triangle closing (bottom right), and preferential attachment (bottom left). The bottom row was not executed due to the tendency of models with random edges to attach all edges to a single node, giving values of the Gini coefficient very close to the theoretical maximum of one.

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of about 51.5%.[6] The pure triangle closing method results in a value of the Gini coefficient of 65.1%, a value similar to (and even superior to) the value in the pure preferential attachment case. Thus, it is indeed the case that a skewed degree distribution is generated by a purely local process of triangle closing, without the need for explicit preferential attachment. We note also that preferential attachment is observed even though the number of nodes in the network ($n = 1,000$) is relatively small when compared to the theoretical model described in the previous section in which the limit $n \to \infty$ is taken.

## IX DISCUSSION

Our experiments have allowed us to observed that triangle closing leads to skewed and scale-free degree distributions. However, the status of a mechanism as *fundamental* is not clear cut. When a phenomenon is explained by another, more fundamental phenomenon, we can consider it as derived. But how can we be sure that a phenomenon is not explained by a more basic phenomenon? What does it mean for a phenomenon to be fundamental? Just as physics cannot declare one theory to be final, we cannot declare one network growth mechanism to be final. Thus, individual instances of triangle closing can for instance be explained by several layers of triangle closing, just as in physics a direct interaction can be explained by a new mediating particle. In the end however, this applies only to specific instances of triangle closing, as it replaces them with other, more detailed instances of triangle closing. Thus, triangle closing *does* play a fundamental role in growing network models, only that it cannot always be derived which three nodes are taking part in it, as one of the three nodes is often hidden. In the end, the only judge of the validity of a model remains the experiment, and in practice, used models do not have to be fundamental – recommenders and information retrieval systems have had enough success by applying preferential attachment directly.

As mentioned in the introduction, triangle closing is itself a general phenomenon that not only applies to pure social networks, but also to other types of networks. In the case of property networks, i.e., networks containing edges between persons and the properties they have, triangle closing can be identified with the concept of homophily, i.e., the concept that friends tend to be similar. As an example, the fact that two smokers become friends can be modelled as the closure of the (person A)–(colleague + smoker)–(person B) triangle, in which "colleague + smoker" is a non-person node of the network representing the property of *being a colleague and a smoker*. Thus, the fact that friends of smokers are more likely to be smokers too (a classical example of homophily) can be analysed as a form of triangle closing in a graph that is not purely a social network, as it contains non-person nodes. Homophily is thus consistent with the view that triangle closing is fundamental (Shalizi and Thomas, 2011).

The problem posed in this paper can be generalised to other graph growth mechanisms. For instance, we may ask whether assortativity (the tendency of connected nodes to have correlated degrees) or community structures emerge from triangle closing alone. In the case of community structures, triangle closing trivially plays a role, as triangle closing by construction leads to tightly connected graphs. As for assortativity, the fact that both assortativity (a positive correlation between degrees) and dissortativity (a negative correlation between degrees) have been observed in social networks points to the fact that a single model such as triangle closing cannot (and is not expected to) explain all properties of a social network, and other phenomena must be at work, which may or may not be local.

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[6]In this and all subsequent cases labelled as *pure*, the method in question has a probability of $p_{\text{tc,pa}} = 10/11$ while a random edge is added with a probability of $p_{\text{r}} = 1/11$.

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